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INFLUENCE OF ELECTRIC FIELD ON THE COOLING GAS FLOW IN A NUCLEAR REACTOR

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Abstract—The present paper deals with slightly ionized gas flow that corresponds to the flow of a nuclear-reactor cooling gas ionized by nuclear radiation. The gas properties are known from experiments carried out in a reactor loop. It is shown that a transverse electric field may influence the gas flow. This influence is quite analogous to that of a gravitational field upon the liquid flow with non-uniformly distributed density in so-called stratified flows.

NOMENCLATURE

x, y,	co-ordinates;
l,	length;
h,	channel height;
ρ,	density;
n, N,	molecular concentration;
n_0	ionization density;
M,	molecular weight;
t,	time;
$\mathbf{w}$, $\mathbf{W}$,	velocity;
w_0,	mean value of velocity;
$\mathbf{J}$	(absolute) mass flux of mixture component;
$\mathbf{I}$,	relative mass flux of mixture component;
p, P,	pressure;
$\mathbf{F}$,	(external) body force;
$\dot{S}_{m,n}$,	viscous stresses tensor;
η,	dynamic viscosity;
ν,	kinematic viscosity;
T,	absolute temperature;
k,	Boltzmann's constant;
u,	internal energy;
$\mathbf{q}$,	heat flux;
D,	diffusion coefficient;
φ,	electric-field potential;
$\mathbf{E}$,	electric-field intensity;
e,	elementary charge;
$\mathbf{j}$,	electric-current density;
μ,	ions mobility;
ϵ,	dielectric constant;
α,	recombination constant;
β,	probability coefficient of negative ions forming;
ψ,	disturbance stream function;
Φ,	amplitude function of ψ;
γ, λ, δ, c,	elementary disturbance characteristics;
Q,	source term in continuity equation;
R,	sink term in continuity equation;
Ri,	Richardson number;
Fr,	Froude number;
Nu,	Nusselt number.

Indices

1,	cations;
2,	anions;
3,	neutral molecules;
e,	free electrons.

INTRODUCTION

A COOLING gas passing through the technological channel of a nuclear reactor is partially ionized under the influence of nuclear radiation. In [1–3] it is shown that by the exploitation of electric properties of the gas it is possible to influence the character of its flow. One of the possible applications of this phenomenon which is worthy of note is the possibility of achieving gas-flow turbulization near the heat-transfer surfaces under the influence of electric field, and thus increasing heat transfer into the gas. The works mentioned above were carried out along these lines. Reference [3] contains the results of experiments carried out directly in the nuclear reactor on an experimental loop, i.e. under conditions very like the real ones. The data obtained, as well as the results of further experiments which are not yet published, make possible the determination of the main properties of the state of gas in the nuclear-reactor cooling channels. On this basis, the considered phenomenon may be explained theoretically.

THE IONIZED STATE OF THE COOLING GAS

The results of cooling-gas ionization calculations, in which the γ-radiation and fast neutrons [1] were considered as the ionization sources, correspond substantially to measured values of the saturated ionization current [3]. On the basis of the results obtained, the ionization level of gas parameters used is not high enough to influence its heat conductivity, viscosity and specific heat. This fact, indeed, has been confirmed experimentally [3]. On the other hand, the volume concentration of ions achieves great values due to the relatively high pressure of the cooling gas. In comparison with the amount of heat transferred by the gas, the radiation energy absorbed in the gas is small. The electric-field effects are achieved at values of field intensity at which the secondary ionization does not occur. The density of passing currents is so small that their magnetic effect may be neglected.

The process of gas ionization by the sources mentioned above is very complex. According to the elementary scheme, the positive ions and the free electrons will be formed under the influence of ionizers; the latter further will adhere to neutral molecules, thus forming negative molecular ions. Moreover, the dissociation of molecules and the formation of atomic ions will occur here to a great extent, as well as the formation of doubly and multiply charged ions, the ageing of ions etc., depending on gas quality [4]. On the one hand there exist no total data on these processes for conditions considered, while on the other it is necessary, for our investigation, to use a very simple model of the ionized gas which would allow quantitative treatment of the hydrodynamic relations.

Therefore, a model of the ionized gas is chosen in which, besides neutral molecules, there are only positive (cations) and negative (anions) molecular singly charged ions. For a gas under atmospheric pressure which is ionized by X-radiation, this model gives results agreeing with experiments (e.g. [10]). We take into account that in our case considerably high electric field intensities will be used as a result of considerably high ionization density, thus the probability of forming negative molecular ions decreases. Moreover, some gases used as reactor coolants (helium, nitrogen, carbon dioxide) have in their pure state a small, or a very small probability of forming negative ions. Therefore, it is necessary to supplement the results obtained for the chosen model of the ionized gas by the analysis of the influence of free electrons.

From the viewpoint of energetics it may be assumed for the chosen gas model that heat sources defined by recombination of ions are located spaciously in the gas. The cations which fall upon the cathode transfer ionization energy to it. According to this concept, affinity of an electron to a neutral molecule and work function of an electron from an electrode are neglected. Since the energy delivered to the gas by radiation is small in comparison with the heat transferred by the gas, and also the intensity of gas ionization is small, we shall not take the influence of ionization energy release into further consideration.

THE GENERAL FLOW EQUATIONS FOR A PARTIALLY IONIZED GAS

On the basis of the above-mentioned assumptions, the present problem may be formulated as the flow of a gas mixture consisting of three components—neutral molecules, positive (cations) and negative (anions) singly charged molecular ions. In some cases the influence of free electrons will be separately taken into account.

BASIC EQUATIONS

As in the dynamics of gas mixtures [5], the concept of the component density ρ_i(g/cm^3) is introduced:

$$\rho_i = n_i M_i \frac{1}{6{\cdot}024 \times 10^{23}}, \tag{1}$$

where n_i is the number of molecules per cubic centimetre (molecular concentration) and M_i

is the molecular weight of a component. The concept "molecular concentration" will be further applied to the electrically active components to express the electric charges and the electric-field dynamic effects.

For the mixture density it is then:

$$\rho = \sum_i \rho_i = \frac{1}{6{\cdot}024 \times 10^{23}} \sum_i n_i M_i. \qquad (2)$$

Let us designate the component mass flux, i.e. the quantity of mass of a component passing through unit area per unit time, as $\mathbf{J}_i$ (g/cm²s). Then, the velocity $\mathbf{w}_i$ (cm/s) of the component is determined by the equation:

$$\mathbf{w}_i = \frac{\mathbf{J}_i}{\rho_i} \qquad (3)$$

and the velocity $\mathbf{w}$ of the mixture by

$$\mathbf{w} = \frac{\mathbf{J}}{\rho} = \frac{\sum_i \mathbf{J}_i}{\sum_i \rho_i} = \frac{\sum_i n_i M_i \mathbf{w}_i}{\sum_i n_i M_i}. \qquad (4)$$

The value

$$\mathbf{I}_i = \rho_i (\mathbf{w}_i - \mathbf{w}) \qquad (5)$$

will be called the relative mass flux of a component.

On the basis of (2) and (4) it is obvious that

$$\sum_i \mathbf{I}_i = 0. \qquad (6)$$

Indices are assigned to individual components as given in the Nomenclature.

CONTINUITY EQUATIONS

For individual components the continuity equation will be

$$\frac{\mathrm{d}\rho_i}{\mathrm{d}t} + \rho_i \operatorname{div} \mathbf{w} + \operatorname{div} \mathbf{I}_i = Q_i - R_i, \qquad (7)$$

where Q_i is the source, i.e. the quantity of a component originating in a volume unit per unit time, and R_i is the sink which stands for vanishing quantity of the component.

For ions in the chosen gas model it will be

$$Q_i = n_0 M_i \frac{1}{6{\cdot}024 \times 10^{23}}, \qquad (8)$$

where n_0 in the ionization density (ions/cm³s) and M_i is the molecular weight in atomic unit of mass, and

$$R_i = a \rho_i n_k, \qquad (9)$$

where a is the recombination constant and n_k the molecular concentration of the second component.

Further, the ionization density is assumed to be constant for the whole chosen region; it depends on radiation intensity, gas quality and gas density. The recombination constant will depend on the gas quality as well, but its dependence upon pressure is, however, more complicated and has no fundamental characteristic.

$$Q_i = \frac{1}{6{\cdot}024 \times 10^{23}} M_i a n_j n_k \qquad (10)$$

and

$$R_i = n_0 M_i \frac{1}{6{\cdot}024 \times 10^{23}}. \qquad (11)$$

Taking equations (8–11) as well as (2) and (6) into account, the sum of equations of type (7) for all the components gives the continuity equation for the mixture:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} + \operatorname{div} \mathbf{w} = 0. \qquad (12)$$

For evaluation of the influence of free electrons, the continuity equation (7) is applied to free electrons, for which the source term is of type (8). The sink term is as follows [4]:

$$R_e = \beta \rho_e + a_e \rho_e n_1, \qquad (13)$$

where the coefficient β represents the probability of anion creation (β depends on field intensity, gas quality and gas pressure, on the concentration of neutral molecules), a_e is the recombination constant for free electrons and n_1 is the molecular concentration of cations.

Consequently, the source term for anions will be

$$Q_2 = \beta n_e M_2 \frac{1}{6{\cdot}024 \times 10^{23}}, \qquad (14)$$

and the sink term for cations:

$$R_1 = a \rho_1 n_2 + a_e \rho_1 n_e. \qquad (15)$$

MOMENTUM EQUATIONS

In the case considered, the momentum equations for a mixture will differ from the Navier–Stokes equations only by the influence of electric field. They may be written in the form:

$$\rho \frac{\mathrm{d}w_x}{\mathrm{d}t} = F_x\rho + (n_1 - n_2)eE_x - \frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \times \left[\eta\left(2\frac{\partial w_x}{\partial x} - \frac{2}{3}\operatorname{div}\mathbf{w}\right)\right] + \frac{\partial}{\partial y}\left[\eta\left(\frac{\partial w_y}{\partial x} + \frac{\partial w_x}{\partial y}\right)\right] + \frac{\partial}{\partial z}\left[\eta\left(\frac{\partial w_z}{\partial x} + \frac{\partial w_x}{\partial z}\right)\right];$$

$$\rho \frac{\mathrm{d}w_y}{\mathrm{d}t} = F_y\rho + (n_1 - n_2)eE_y - \frac{\partial p}{\partial y} + \frac{\partial}{\partial y} \times \left[\eta\left(2\frac{\partial w_y}{\partial y} - \frac{2}{3}\operatorname{div}\mathbf{w}\right)\right] + \frac{\partial}{\partial z} \times \left[\eta\left(\frac{\partial w_z}{\partial y} + \frac{\partial w_y}{\partial z}\right)\right] + \frac{\partial}{\partial x}\left[\eta\left(\frac{\partial w_x}{\partial y} + \frac{\partial w_y}{\partial x}\right)\right]; \quad (16)$$

$$\rho \frac{\mathrm{d}w_z}{\mathrm{d}t} = F_z\rho + (n_1 - n_2)eE_z - \frac{\partial p}{\partial z} + \frac{\partial}{\partial z} \times \left[\eta\left(2\frac{\partial w_z}{\partial z} - \frac{2}{3}\operatorname{div}\mathbf{w}\right)\right] + \frac{\partial}{\partial x} \times \left[\eta\left(\frac{\partial w_x}{\partial z} + \frac{\partial w_z}{\partial x}\right)\right] + \frac{\partial}{\partial y}\left[\eta\left(\frac{\partial w_y}{\partial z} + \frac{\partial w_z}{\partial y}\right)\right].$$

In these equations e denotes the elementary charge, $\mathbf{E}$ the electric field intensity. Other symbols have their usual meaning.

$$\mathbf{E} = -\operatorname{grad}\varphi \quad (17)$$

where φ is the potential of an electric field.

ENERGY EQUATION

Under the given conditions the following formula may be deduced for the energy equation for the flow of a partially ionized gas (when the energy of ionization is neglected):

$$\rho\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{1}{2}w^2 + u + \frac{1}{\rho}\varphi(n_1 - n_2)e\right] = -\operatorname{div}\left[p\mathbf{w} + \mathbf{q} + \mathbf{j}\varphi - \mathbf{w}\dot{S}\right] + \rho\,\mathbf{F}\,\mathbf{w} + (n_1 - n_2)e\frac{\partial\varphi}{\partial t}. \quad (18)$$

The first term on the right-hand side of the equation is the divergence of the energy flow, consisting of the pressure, heat, electrical and viscous forces of energy flow. Symbol $\mathbf{j}$ denotes the density of electric current passing through a gas element

$$\mathbf{j} = 6{\cdot}024 \times 10^{23}\, e\left(\frac{1}{M_1}\mathbf{I}_1 - \frac{1}{M_2}\mathbf{I}_2\right), \quad (19)$$

$\mathbf{q}$ is the heat flux, $\mathbf{w}\dot{S}$ denotes the vector with the components $w_m\dot{S}_{m,\,n}$ where $\dot{S}_{m,\,n}$ is the tensor of viscous stresses.

TRANSFER OF COMPONENTS AND ENERGY

For the given gas model the general equations for the mixture-component fluxes and for the energy flux may be deduced with the help of thermodynamics of irreversible processes. Suppose that relative concentrations of charged components are very small; the molecular weights of components are equal, the components behave themselves as ideal gases, and if the ionization energy is neglected, it is possible to change the complex relations into the simple, known formula:

$$\mathbf{I}_i = -D_i \operatorname{grad}\rho_i \mp \rho_i D_i \frac{e}{Tk}\operatorname{grad}\varphi \quad (20)$$

for the flux of anions and cations. The expression $\pm D_i\ (e/kT)$ is usually replaced by μ_i/p where p is the absolute pressure of the mixture, μ_i is the so-called ionic mobility of the corresponding sign (positive for cations, negative for anions); its value is determined empirically. Thus, we can distinguish the diffusion component and the drift component of the ionic flux, represented by the first and the second term of the right-hand side of equation (20).

The simplifying assumptions applied to flux equations lead to the following phenomena being neglected: interaction of ionic fluxes, baro-diffusion of ions, thermo-diffusion of ions as well as influence of ion transfer upon the heat flux. For the heat flux, then, the same equation as in the case of non-ionized gas is valid.

For evaluating the influence of free electrons under the given conditions we shall apply the following **transfer equation**:

$$\mathbf{I}_e = \rho_e\mathbf{w}_e \quad (21)$$

where $\mathbf{w}_e$ depends on gas quality, its absolute pressure, temperature and the intensity of the electric field. The dependence is of an empirical character. A qualitative representation of it may be obtained on the basis of the elementary kinetic theory of gases. In an analogous way, it is possible to evaluate also the influence of the free electrons on heat-energy transfer. It can be found that for the given conditions this effect may be neglected.

ELECTRIC FIELD

As was mentioned above, under the usual conditions, the ionization level of the cooling gas in a nuclear-reactor channel has a comparatively low value, so that it does not influence the thermophysical gas constants. However, the volume concentration of ions reaches considerably high values because of the high cooling-gas working pressure. For this reason, therefore, the problem is complicated, owing to the distortion of the electric field, caused by a space charge, the influence of which is determined by the equation:

$$\text{div } \mathbf{E} = \frac{4\pi e}{\epsilon}(n_1 - n_2) \tag{22}$$

where ϵ is the gas dielectric constant. At the pressures used it may be taken as: $\epsilon \approx 1$.

TWO-DIMENSIONAL ISOTHERMAL LAMINAR-DEVELOPED GAS FLOW IN A COOLING CHANNEL

To simplify further discussions the two-dimensional, isothermal, laminar-developed flow along the channel of the reactor is considered. We direct the X-axis along the channel wall (Fig. 1). The concept of the developed flow, extended also on the transfer part of the problem, means that the transverse velocity component of the mixture is equal to zero, and values w_x, $\mathbf{w}_i$, ρ_i, $\mathbf{E}$ and φ do not change along the X-axis. A problem so formulated corresponds to the real conditions in a reactor channel with an approximation given merely by the basic assumptions of laminarity and isothermality. Further, the mixture density is considered to be constant. With such assumptions the problem is correspondingly simplified. With the analysis of dynamic equations in the form:

$$\frac{\partial p}{\partial x} = \eta \frac{\partial^2 w_x}{\partial y^2}, \tag{23}$$

$$\frac{\partial p}{\partial y} = (n_1 - n_2)eE_y = neE_y, \tag{24}$$

where $n = (n_1 - n_2)$, it is obvious that there takes place a flow with a transverse pressure

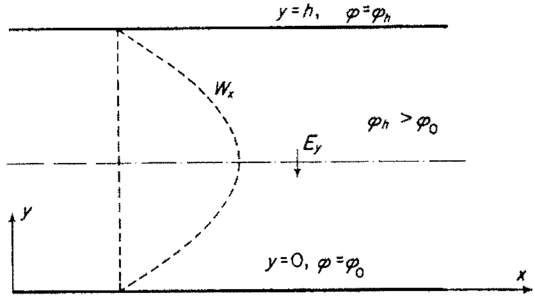

FIG. 1.

gradient. The value and sign of the pressure gradient will depend on the distribution of concentration of ions and on the electric potential drop between the channel walls.

Depending on the reciprocal distribution of ion concentration and the electric field intensity, the flow considered will be in a state of stable or unstable equilibrium. Let us analyse the case illustrated in Fig. 2. At the distribution of ion concentration in case (a), a gas particle will be in equilibrium with the surrounding medium if its y co-ordinate does not change. At a casual displacement of the particle, the force (nE_y) effecting it will exceed the lift of surrounding particles, and the casual disturbance will be amplified. In the second case (b), the force (nE_y) will be smaller than the lift of the surrounding medium and the disturbance will be damped out. As is seen from Fig. 2, these effects occur at disturbances of both signs. The case of unstable equilibrium corresponds to different signs of E_y and dn/dy, i.e. $E_y(dn/dy) < 0$. On the other hand, $E_y(dn/dy) > 0$ characterizes the stable equilibrium.

Quantitative indications of stabilization or

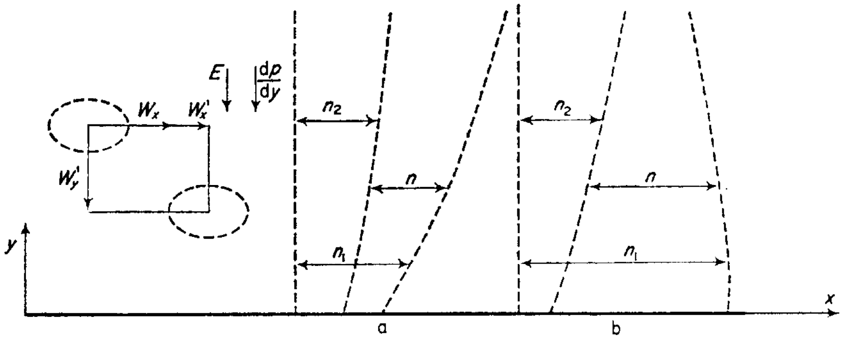

FIG. 2.

turbulence of the flow may be obtained by the method of small disturbances, In the given case, in order to simplify the use of this method, the following assumptions are taken.

(i) The disturbances do not influence the distribution of the electric-field intensity. This assumption is valid at the negligibly small distortion of the electric field caused by the space charge. When the space charge does effect the electric-field intensity, we may consider —on account of the small dimensions of disturbances—its influence as a small value of the second order and neglect it.

(ii) In disturbances, drift and diffusion of ions will be neglected.

Let us substitute values

$$\begin{aligned} N_i(x, y, t) &= n_i(y) + n_i'(x, y, t); \\ W_x(x, y, t) &= w_x(y) + w_x'(x, y, t); \\ W_y(x, y, t) &= w_y'(x, y, t), \end{aligned} \tag{25}$$

(where N_i, W_x and W_y are concentrations and velocities in the disturbed flow; n_i, w_x, $w_y = 0$ are corresponding values in the undisturbed flow and n_i', w_x', w_y' in disturbed flow) into equation (7) rearranged for our case. After neglecting terms of disturbances of a higher order, we obtain the following equation:

$$\left.\begin{aligned} &\frac{\partial n_i'}{\partial t} + w_x\frac{\partial n_i'}{\partial x} + w_y'\frac{\partial n_i}{\partial y} - D_i\frac{\partial^2 n_i}{\partial y^2} - D_i\frac{\partial^2 n_i'}{\partial y^2} \\ &- D_i\frac{\partial^2 n_i'}{\partial x^2} + \frac{\mu_i}{p}\frac{\partial}{\partial y}(E_y n_i) + \frac{\mu_i}{p}\frac{\partial}{\partial y}(E_y n_i') \\ &= n_0 - a n_i n_k - a n_i' n_k - a n_i n_k'. \end{aligned}\right\} \tag{26}$$

For undisturbed flow the following is valid:

$$-D_i\frac{\partial^2 n_i}{\partial y^2} + \frac{\mu_i}{p}\frac{\partial}{\partial y}(E_y n_i) = n_0 - a n_i n_k. \tag{27}$$

From (26) and (27), the continuity equation of disturbed motion of ions is obtained in the form:

$$\begin{aligned} &\frac{\partial n_i'}{\partial t} + w_x\frac{\partial n_i'}{\partial x} + w_y'\frac{\partial n_i}{\partial y} - D_i\frac{\partial^2 n_i'}{\partial y^2} - D_i\frac{\partial^2 n_i'}{\partial x^2} \\ &+ \frac{\mu_i}{p}\frac{\partial}{\partial y}(E_y n_i') = -a n_i' n_k - a n_i n_k'. \end{aligned} \tag{28}$$

In equation (28), the diffusion flux along the X-axis may be neglected in comparison with the convective flux along the same axis. Taking into account that the disturbance velocity w_y' is of the same order as the velocity of the basic flow, then the diffusion flux component along the Y- axis may be neglected in comparison with the convection flux component. If we further neglect also the drift component of ion motion, we obtain the equation:

$$\frac{\partial(n_1' - n_2')}{\partial t} + w_x\frac{\partial(n_1' - n_2')}{\partial x} + w_y'\frac{\partial(n_1 - n_2)}{\partial y} = 0, \tag{29}$$

by subtracting simplified equations (28) for ions of both signs. Equation (29) will substantially simplify further operations.

The neglect of the drift component in disturbances will be undoubtedly valid in the case of small values of the electric-field intensity. But at the intensity values which occur in practice, the drift velocity may compare with the velocity of the basic flow and even exceed it. Therefore, equation (29) is valid for a general case only as a simplifying assumption.

STABILIZATION AND TURBULENCE OF FLOW BY ELECTRIC FIELD

If we take into account the assumptions mentioned above, the problem of stability of the given flow may be solved using the method of disturbances. In addition to equations (25) we introduce

$$P(x, y, t) = p(x, y) + p'(x, y, t) \tag{30}$$

where P is the pressure in the disturbed flow, p is the pressure in the undisturbed flow and p' is the pressure disturbance. The disturbance quantities are expressed in the usual form [6]:

$$\left.\begin{aligned} w_x'(x, y, t) &= w_x^+(y)\, e^{i(\gamma x - \delta t)}; \\ w_y'(x, y, t) &= w_y^+(y)\, e^{i(\gamma x - \delta t)}; \\ p'(x, y, t) &= p^+(y)\, e^{i(\gamma x - \delta t)}; \\ n'(x, y, t) &= n_1'(x, y, t) - n_2'(x, y, t) \\ &= n^+(y)\, e^{i(\gamma x - \delta t)}. \end{aligned}\right\} \tag{31}$$

where γ is a real number and determines the wavelength of a disturbance as $\lambda = 2\pi/\gamma$, δ is a complex number (its real part is the cyclic frequency of a disturbance) and the imaginary part is the amplification coefficient of a disturbance (its positive value corresponds to amplification, the negative one to damping). We make the usual assumption that each disturbance appearing in the flow may be expanded in a Fourier series as a sum of elementary disturbances of the type mentioned, and the stability of the given flow may be investigated for individual elementary disturbance.

The disturbed flow will be described by the general equations for the two-dimensional flow arranged for the particular case:

$$\rho\left(\frac{\partial W_x}{\partial t} + W_x\frac{\partial W_x}{\partial x} + W_y\frac{\partial W_x}{\partial y}\right) = -\frac{\partial P}{\partial x} + \eta\left(\frac{\partial^2 W_x}{\partial x^2} + \frac{\partial^2 W_x}{\partial y^2}\right) \tag{32}$$

$$\rho\left(\frac{\partial W_y}{\partial t} + W_x\frac{\partial W_y}{\partial x} + W_y\frac{\partial W_y}{\partial y}\right) = -\frac{\partial P}{\partial y} + \eta\left(\frac{\partial^2 W_y}{\partial x^2} + \frac{\partial^2 W_y}{\partial y^2}\right) + (N_1 - N_2)eE_y \tag{33}$$

$$\frac{\partial W_x}{\partial x} + \frac{\partial W_y}{\partial y} = 0. \tag{34}$$

Substituting relations (25) and (30) into equations (32) and (34), rearranging them with regard to (23) and (24) and neglecting terms of disturbances of a higher order, we obtain

$$\rho\left(\frac{\partial w_x'}{\partial t} + w_x\frac{\partial w_x'}{\partial x} + w_y'\frac{\partial w_x}{\partial y}\right) = -\frac{\partial p'}{\partial x} + \eta\left(\frac{\partial^2 w_x'}{\partial x^2} + \frac{\partial^2 w_x'}{\partial y^2}\right) \tag{35}$$

$$\rho\left(\frac{\partial w_y'}{\partial t} + w_x\frac{\partial w_y'}{\partial x}\right) = -\frac{\partial p'}{\partial y} + \eta\left(\frac{\partial^2 w_y'}{\partial x^2} + \frac{\partial^2 w_y'}{\partial y^2}\right) + n'eE_y \tag{36}$$

$$\frac{\partial w_x'}{\partial x} + \frac{\partial w_y'}{\partial y} = 0. \tag{37}$$

This system of equations is completed by the modified equation (29):

$$\frac{\partial n'}{\partial t} + w_x\frac{\partial n'}{\partial x} + w_y'\frac{\partial n}{\partial y} = 0. \tag{38}$$

To simplify calculations we shall eliminate the variable p' from equations (35) and (36) and obtain

$$\frac{\partial^2 w_x'}{\partial t\partial y} - \frac{\partial^2 w_y'}{\partial t\partial x} + \frac{\partial w_x}{\partial y}\frac{\partial w_x'}{\partial x} + w_x\left(\frac{\partial^2 w_x'}{\partial x\partial y} - \frac{\partial^2 w_y'}{\partial x^2}\right) + \frac{\partial w_y'}{\partial y}\frac{\partial w_x}{\partial x} + w_y'\frac{\partial^2 w_x}{\partial y^2} = \nu\left(\frac{\partial^3 w_x'}{\partial x^2\partial y} + \frac{\partial^3 w_x'}{\partial y^3} - \frac{\partial^3 w_y'}{\partial x^3} - \frac{\partial^3 w_y'}{\partial x\partial y^2}\right) - \frac{e}{\rho}E_y\frac{\partial n'}{\partial x}. \tag{39}$$

After substituting equations (31) into equations (37–39) we get

$$i\gamma w_x^+ = -\frac{\partial w_y^+}{\partial y} \tag{40}$$

$$in^+(\gamma w_x - \delta) = -w_y^+\frac{\partial n}{\partial y} \tag{41}$$

$$(\gamma w_x - \delta)\left(i\frac{\partial w_x^+}{\partial y} + \gamma w_y^+\right) + w_y^+ \frac{\partial^2 w_x}{\partial y^2}$$
$$= i\nu\left[\gamma^2\left(i\frac{\partial w_x^+}{\partial y} + \gamma w_y^+\right)\right.$$
$$\left. - \frac{\partial^2}{\partial y^2}\left(i\frac{\partial w_x^+}{\partial y} + \gamma w_y^+\right)\right] - i\gamma\frac{e}{\rho}E_y n^+. \qquad (42)$$

Equations (40–42) represent a system for variables w_x^+, w_y^+ and n^+. Let us introduce the stream function of the disturbance flow in the following form:

$$\psi(x, y, t) = \Phi(y)\, e^{i(\gamma x - \delta t)}$$

i.e.

$$w_x' = \frac{\partial\psi}{\partial y}, \qquad w_y' = -\frac{\partial\psi}{\partial x},$$

or

$$w_x^+ = \frac{\mathrm{d}\Phi}{\mathrm{d}y}, \qquad w_y^+ = -i\gamma\Phi,$$

into the equation system (40–42).

We obtain the basic equation for the disturbance flow in its ordinary form:

$$(w_x - c)\left(\frac{\mathrm{d}^2\Phi}{\mathrm{d}y^2} - \gamma^2\Phi\right) - (w_x - c)\Phi\frac{\partial^2 w_x}{\partial y^2}$$
$$+ \Phi\frac{e}{\rho}E_y\frac{\partial n}{\partial y} = \frac{i\nu}{\gamma}(w_x - c)\left[\gamma^2\left(\frac{\mathrm{d}^2\Phi}{\mathrm{d}y^2} - \gamma\Phi\right)\right.$$
$$\left. - \frac{\mathrm{d}^2}{\mathrm{d}y^2}\left(\frac{\mathrm{d}^2\Phi}{\mathrm{d}y^2} - \gamma^2\Phi\right)\right], \qquad (43)$$

where

$$c = \frac{\delta}{\gamma}.$$

The real part of c represents the phase velocity of a disturbance; the imaginary part of c is the amplification coefficient.

Substituting $w_x = w_0 \,.\, \mathbf{w}_x$, $c = w_0 \,.\, \mathbf{c}$, $y = l \,.\, \mathbf{y}$ and $\gamma = (1/l) \,.\, \boldsymbol{\gamma}$ where l is the characteristic dimension (length) and w_0 is the characteristic velocity in the region considered, we transform equation (43) to a dimensionless form and obtain

$$(\mathbf{w}_x - \mathbf{c})\left(\frac{\mathrm{d}^2\Phi}{\mathrm{d}\mathbf{y}^2} - \boldsymbol{\gamma}^2\Phi\right) - (\mathbf{w}_x - \mathbf{c})\Phi\frac{\mathrm{d}^2\mathbf{w}_x}{\mathrm{d}\mathbf{y}^2}$$
$$+ \frac{eE_y l(\mathrm{d}n/\mathrm{d}\mathbf{y})}{\rho w_0^2}\Phi = i\frac{\nu}{w_0 \,.\, l}\frac{1}{\boldsymbol{\gamma}}(\mathbf{w}_x - \mathbf{c})$$
$$\left[\boldsymbol{\gamma}^2\left(\frac{\mathrm{d}^2\Phi}{\mathrm{d}\mathbf{y}^2} - \boldsymbol{\gamma}^2\Phi\right) - \frac{\mathrm{d}^2}{\mathrm{d}\mathbf{y}^2}\left(\frac{\mathrm{d}^2\Phi}{\mathrm{d}\mathbf{y}^2} - \boldsymbol{\gamma}^2\Phi\right)\right]. \qquad (44)$$

For the investigation of the problem of a particular region of the flow equation (44) is completed by boundary conditions determining the behaviour of disturbances on the flow boundaries. Solving this problem for a given wavelength as an eigenvalue problem, we determine both the function $\Phi(y)$ and the complex value of c characterizing the phase velocity and the amplification of the elementary disturbance.

Analysing equation (44), we may determine that its real part is analogous to the real part of the equation of the disturbance motion for the so-called stratified flow [7] in the case when the inertia effect of stratification may be neglected, i.e. when the Froude number is considerably smaller than unity. The expression

$$\frac{eE_y(\mathrm{d}n/\mathrm{d}y)}{\rho(w_0/l)^2} \qquad (45)$$

is analogous to the Richardson number for the stratified flow.

The imaginary part of equation (44) is identical with that of the equation for the disturbance flow in the case of the simple flow.

For a given field of $n(y)$, $E_y(y)$ and $w_x(y)$ and for given boundary conditions, the problem of stability of the flow investigated and of its parts may be solved by known methods.

It is possible to use the analogy with the problem of the stratified flow for $Fr \ll 1$. This analogy is based on the identity of the real part of the disturbance equation, and further on the fact that the influence of stratification on the solution of the disturbance-equation real part (i.e. on flow without friction) is considerably greater than on the particular solution which represents the influence of friction [7].

CHARGE AND ELECTRIC-FIELD INTENSITY DISTRIBUTION IN A CHANNEL

For the considered case of developed flow in a channel, equation (7) will be as follows:

$$\frac{\mathrm{d}}{\mathrm{d}y}(I_i) = Q_i - R_i. \qquad (46)$$

Since the dynamic effects of the mass transfer are not considered here, molecular concentrations instead of densities are substituted into equations (8), (9) and (20). Substituting these relations into equation (46) we get:

$$\frac{d}{dy}\left(-D_i\frac{dn_i}{dy}+\frac{\mu_i}{p}n_iE\right)=n_0-an_in_k. \quad (47)$$

Equation (47) describes the concentration distribution of ions of both signs in the channel with regard to the corresponding sign of μ_i. Boundary conditions are:

(i) Condition of zero ion concentration on the electrode of the opposite sign, i.e. in the given co-ordinate system:

$$\begin{aligned} y&=0, & n_1&=0,\\ y&=h, & n_2&=0. \end{aligned} \quad (48)$$

(ii) Condition of zero flux on the electrode of the same sign, i.e. (further, μ_1 and μ_2 are absolute values of ionic mobilities):

$$\begin{aligned} -D_1\frac{dn_1}{dy}+\frac{\mu_1}{p}n_1E&=0 \text{ for } y=h\\ -D_2\frac{dn_2}{dy}-\frac{\mu_2}{p}n_2E&=0 \text{ for } y=0. \end{aligned} \quad (49)$$

The problem is completed by the dependence of the electric-field intensity on the space charge, according to equation (22):

$$\frac{dE}{dy}=4\pi e(n_1-n_2) \quad (50)$$

and by boundary conditions determining values of the potential on the electrodes.

The problem described by equations (47–50) is identical with that of electric current in a flat condenser with ionized gas, which has been solved with various approximations [8–11]. Some basic facts are taken from these sources.

The diffusion component of ion migration essentially influences the distribution of ion concentration near the electrode of opposite sign. In this region, which may be called a diffusion layer, the gradient of the ion concentration reaches great values. From this point of view, the gradient of the ion concentration on a surface of the electrode of opposite sign may be a criterion characterizing conditions in the diffusion layer. On the basis of (47–49) the following should be valid for this gradient:

$$\left.\frac{dn_1}{dy}\right|_{y=0}=\frac{\mathbf{j}}{eD_1}; \qquad \left.\frac{dn_2}{dy}\right|_{y=h}=-\frac{\mathbf{j}}{eD_2} \quad (51)$$

where $\mathbf{j}$ is the density (absolute value) of passing electric current. In the case of saturated current:

$$\mathbf{j}=n_0eh.$$

The value of the maximum gradient of concentration of negative ions will decrease, since some current is transferred by free electrons, thus the effective value of $\mathbf{j}$ in (51) will be correspondingly smaller.

In the region outside the diffusion layer the gradient of the ion concentration will be determined mainly by the drift component of their transfer. Until values of the electric-field intensity are small, those of the gradient of the ion concentration outside the diffusion layer will always be smaller than in the diffusion layer. In the case of saturated current and negligibly low influence of the space charge ($E=\text{const.}$) in the presence of the drift component alone, it will be

$$\begin{aligned} \frac{dn_1}{dy}&=\frac{n_0p}{\mu_1E}\\ \frac{dn_2}{dy}&=-\frac{n_0p}{\mu_2E}. \end{aligned} \quad (52)$$

In the given co-ordinate system E is negative.

The absolute value of the gradient evaluated from equation (52) is greater than the real value because of the influence of the diffusion component of ion migration and in the case of non-saturated current, because of the influence of recombination. The influence of the space charge will have various signs in different regions of the channel.

As a result of the small height of diffusion layers, the distribution of the electric-field intensity is determined mainly by the space charge in the central region of the channel. Inside the channel the minimum absolute value of the field intensity will be achieved at $n_1=n_2$.

at the point, the position of which will be determined by mutual values of ion mobilities. As a result of the presence of the influence of free electrons, this point will move to the anode. From the minimum point to boundaries of diffusion layers, the absolute value of the field intensity increases uniformly in both directions. On the boundaries of the diffusion layer there is an inflexion point of the curve E, so that the condition

$$\frac{dE}{dy} \simeq 0$$

is approximately fulfilled on both electrodes.

On both electrodes the values of E are again determined by mutual values of ion mobilities. The absolute intensity value on the anode diminishes under the influence of free electrons.

As a result of the non-uniform distribution of the electric field intensity, the absolute value of the gradient of the ion concentration in the central region increases towards the electrode of the same sign and decreases towards the electrode of opposite sign.

The distribution of the ion concentrations and of the electric-field intensity in the central region and in the diffusion layer is plotted on Fig. 3.

INFLUENCE OF THE ELECTRIC FIELD ON GAS FLOW

Considering the electric-field influence on the gas flow, we may divide the channel into the region of diffusion layers near electrodes and the central region of the channel.

In the diffusion layers for the given co-ordinate system and given values of the field intensity, the influence will be

$$\frac{dn}{dy} > 0$$

and, because $E < 0$, the electric field will have a turbulent effect on the flow. The state in the diffusion layer may be characterized by the gradient of the concentration of opposite-sign ions on the electrode surface (at field intensities used, the influence of the gradient of the concentration of ions of the same sign is very small) and by the value E on the electrode or on the boundary of the diffusion layer. The

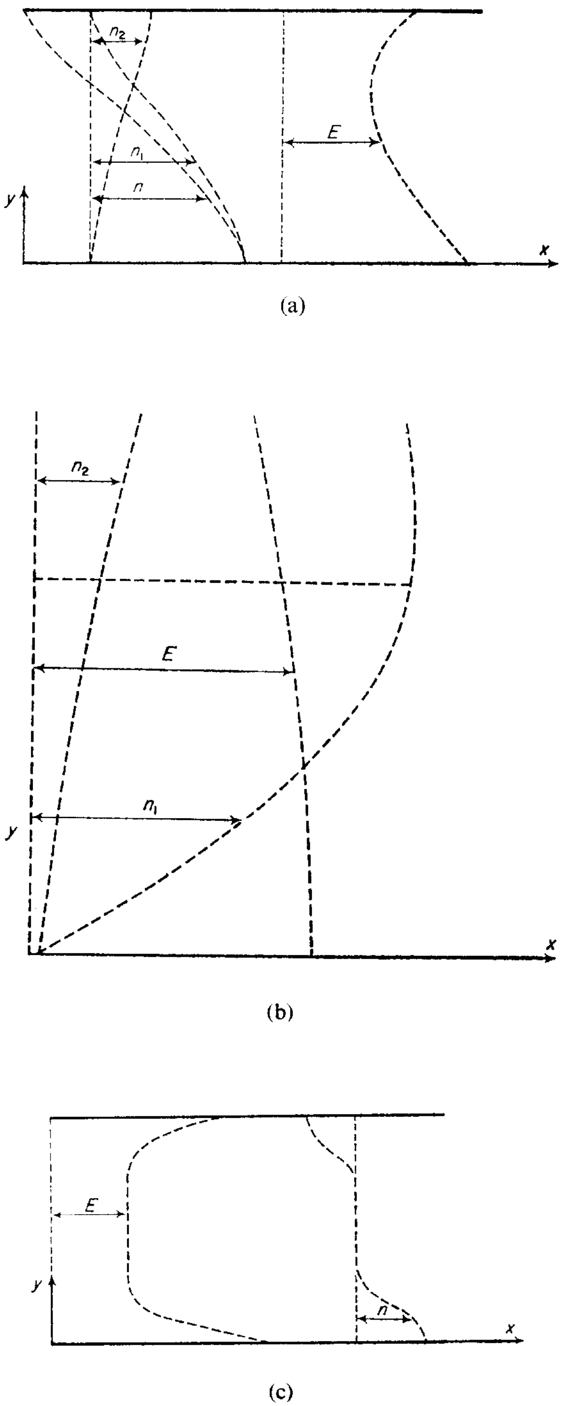

FIG. 3 (a, b and c).

dependence of both of these characteristic values on gas quality and parameters and on the other quantities may be deduced from the works mentioned above.

Taking into account the described state in the diffusion layer near the anode, we may expect that the turbulence influence for the gases investigated will be considerably smaller there than in the diffusion layer near the cathode. This fact was confirmed experimentally [3].

Turbulence of the gas flow near the electrode which is a heat-transfer surface causes heat-transfer intensification near this surface. The heat-transfer intensification may reach considerable values and was also observed outside the laminar and transient regions of the flow in a channel [3]. The experimental dependence of the ratio of the Nusselt numbers obtained at the same values *Re* in a reactor-cooling channel in the presence and absence of a transverse electric field is graphically plotted in Fig. 4. Values obtained in the absence of the electric field are identical with values without an external source of gas ionization; this corresponds to the assumption of low ionization level. Experiments were carried out with air at atmospheric pressure in the velocity range 4–23 m/s, voltage 1000–3000 V and reactor power 200–2000 kW. The negative electrode was the heat-transfer surface. As is seen from the figure, the ratio Nu'/Nu under considered conditions (i.e. in a comparatively narrow region of the change of variables) is a single-value function of the expression $V_R\varphi/w_0^2$, where V_R is the reactor output which corresponds to the ionization density in gas, φ is the difference of electric potential between the channel walls, and w_0 is the average velocity in the channel.

In the flow core the influence will be:

$$\frac{dn}{dy} < 0$$

since $E < 0$, and so the electric field will have the stabilizing influence there. For the field intensities used, this influence will be essentially weaker than the inverse influence in the diffusion layer. Usually the gradient of space charge inside the channel is not constant, and the position of its maximum depends to a great extent on gas parameters as well as on the saturation ratio of passing current. At low currents it approaches the boundary of the diffusion layer (Fig. 3c). At very low currents, i.e. at very low field intensities, it may happen that the stabilizing influence will prevail even near the electrode. Such a case is characterized by a decrease of heat-transfer intensity near the wall in comparison with conditions without the influence of the electric field. This fact was observed in some of the latest experiments, which have not yet been published.

CONCLUSIONS

In technological channels of the nuclear reactor, the cooling gas is partially ionized under the influence of radiation. The level of its ionization is not high enough to influence its thermophysical properties. As a result of the high working gas pressure, the volume concentration of ions is considerable. This makes it possible to influence the gas flow by creation of an electric field in the channel.

The mechanism of this influence on the gas flow through a transverse electric field is

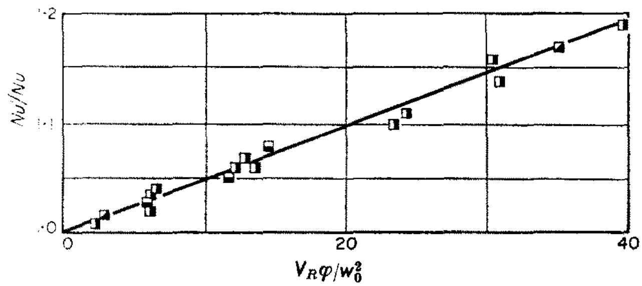

FIG. 4.

analogous to the case of the so-called stratified flow for:

$$Ri = \frac{eE(\mathrm{d}n/\mathrm{d}y)}{\rho(w_0/l)^2}, \qquad Fr \ll 1.$$

By means of relations for electric current in a condenser with ionized gas, the problem of stability of the developed flow in a channel may be solved for individual cases by the method of small disturbances.

A strong gas turbulence may be achieved by a suitable choice of parameters. When heat-transfer surfaces are electrodes, an increase of heat transfer results.

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Résumé—Cet article concerne l'écoulement d'un gaz faiblement ionisé, correspondant à l'écoulement du réfrigérant dans les réacteurs nucléaires, qui est ionisé par le rayonnement atomique. Les propriétés du gaz sont connues à partir des expériences faites dans un réacteur. On démontre qu'un champ électrique transversal peut avoir une influence sur l'écoulement. Cette action est tout à fait analogue à celle d'une champ gravitationnel sur l'écoulement d'un liquide avec distribution de densité: écoulements dits stratifiés.

Zusammenfassung—Die Arbeit behandelt die Strömung schwach ionisierter Gase, wie sie infolge radioaktiver Strahlung bei der Kühlung von Kernreaktoren vorkommen. Aus Versuchen in einer Reaktorschleife ergaben sich die Stoffwerte des Gases. Die Gasströmung kann durch ein quer dazu angelegtes elektrisches Feld beeinflusst werden. Dieser Einfluss ist analog dem des Schwerefeldes auf die Flüssigkeitsströmung bei ungleicher Dichteverteilung, die sogenannte Schichtströmung.

Аннотация—В работе рассматривается течение малоионизированного газа, что соответствует охлаждающему газу в ядерном реакторе, ионизирующемуся ядерным излучением. Свойства газа получены на основании экспериментов, которые проводились в реакторной петле. Показано, что поперечным электрическим полем можно оказывать воздействие на течение газа аналогичное влиянию гравитационного поля на течение жидкости с неравномерно распределенной плотностью в так называемых слоистых течениях.